Kernel Bounds for Path and Cycle Problems

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Joint work with
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Universiteit Utrecht

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Path and Cycle problems

Path and Cycle problems

Long Path

- Given G and an integer ℓ, does G contain a path on at least ℓ vertices?

Long Cycle

- Given G and an integer ℓ, does G contain a cycle on at least ℓ vertices?

Disjoint Paths

- Given G and pairs of vertices $(s_1, t_1), \dots, (s_\ell, t_\ell)$, are there vertex-disjoint paths connecting each s_i to t_i?

Disjoint Cycles

- Given G and an integer ℓ, are there ℓ vertex-disjoint simple cycles in G?

Background

- Various path and cycle problems have been important to the development of parameterized complexity

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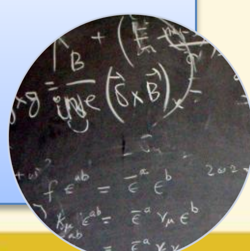
- **Disjoint Paths** lies at the heart of the Graph Minors algorithm
- **Long Path** was one of the first problems known to be fixed-parameter tractable
- **Long Path** was one of the main motivations for the kernel lower-bound framework
- **Disjoint Cycles** inspired one of the first non-trivial compositions

Theoretical



- **Long Path** has applications in computational biology
- …

Practical

Previous results

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Table 1. k-path in time $O^*(f(k))$

$k!$		Monien [29]	12.6^k		Chen *et al.* [6]
$k!2^k$		Bodlaender [5]	4^k	r	Chen *et al.* [6]
5.44^k	r	Alon *et al.* [1]	2.83^k	r	Koutis (2008) [21]
c^k		$c > 8000$, Alon *et al.* [1]	2^k	r	Williams [37]
16^k		Kneis *et al.* [25]	1.66^k	r	*this paper*

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- Natural parameterizations k-Path, k-Disjoint Paths, k-Disjoint Cycles are fixed-parameter tractable but do not admit polynomial kernels unless NP ⊆ coNP/poly [BodlaenderDFH@ICALP'08, BodlaenderTY@ESA'09, Robertson&Seymour]

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  - For k-Path: not even a polynomial kernel on connected planar graphs [ChenFM@CiE'09]

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- How to guide the search for good reduction rules?
  - Non-standard parameters!

- One example known:

Hamiltonian Cycle parameterized by Max Leaf Number has a kernel with 5.75k vertices [FellowsLMMRS@CiE'07]

Our results

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**Long Path, Long Cycle,
Disjoint Paths, Disjoint Cycles**

- Admit $O(k^2)$-vertex kernels parameterized by Vertex Cover Number
- Admit polynomial kernels parameterized by Max Leaf Number

Long Path & Long Cycle

- Admit polynomial kernels parameterized by vertex-deletion distance to a Cluster graph

Hamiltonian Path & Hamiltonian Cycle

- Do not admit polynomial kernels parameterized by vertex-deletion distance to an outerplanar graph

Path problems with Forbidden Pairs

- First study of parameterized complexity: para-NP-completeness, FPT, W[1]-hardness and kernel lower-bounds

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Generalizes kernel for Hamiltonian Cycle by [FellowsLMMRS@CIE'07]

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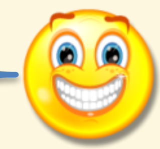

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	vc(G)	vc(H)	tw(H)	tw(G ∪ H)	vc(G ∪ H)
$s-t$ Path F.P.	W[1]-hard	FPT	Para-NP-c	FPT	No poly
Shortest $s-t$ Path F.P.	W[1]-hard	FPT	Para-NP-c	FPT	No poly
Longest $s-t$ Path F.P.	W[1]-hard	Para-NP-c	Para-NP-c	FPT	No poly
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Quadratic-vertex kernel parameterized by Vertex Cover #

LONG CYCLE

Quadratic-vertex kernel for Long Cycle by Vertex Cover

- Input: Graph G, vertex cover X of G, integer ℓ
- Question: Does G have a cycle on at least ℓ vertices?

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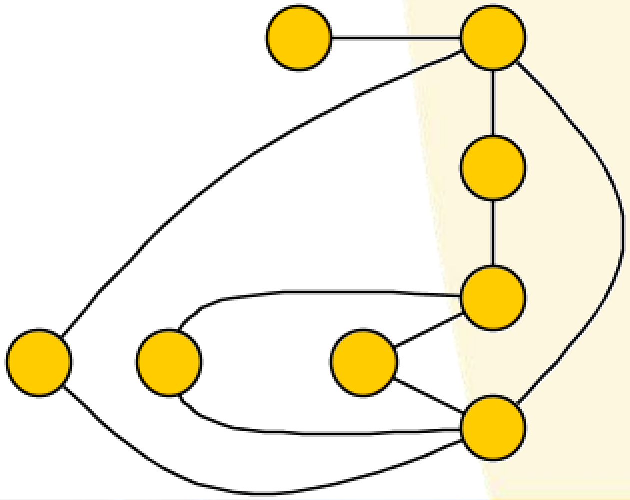

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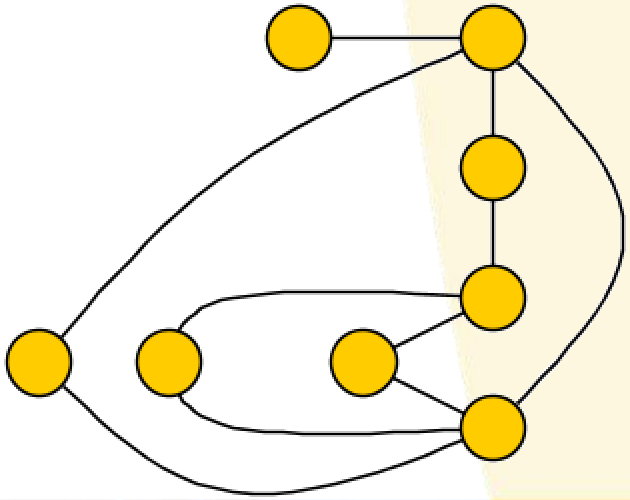
- Example for $\ell = 6$



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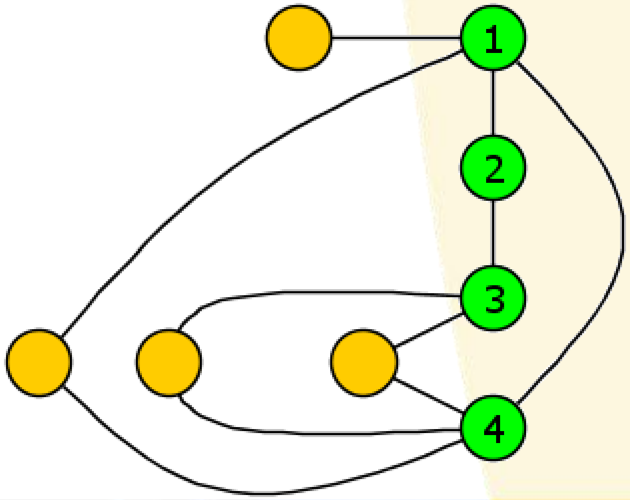

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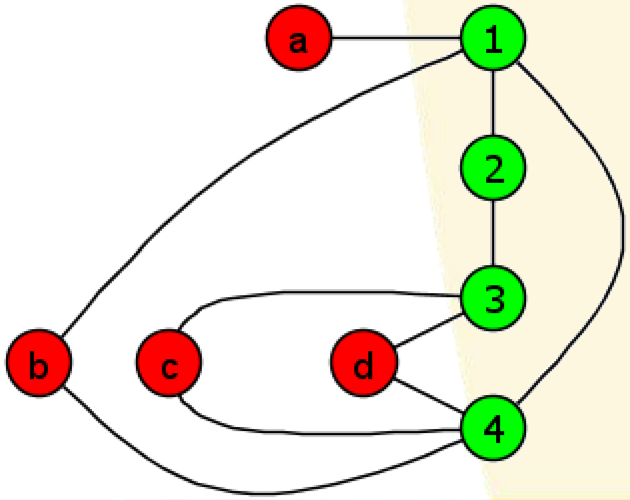

Quadratic-vertex kernel for Long Cycle by Vertex Cover

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  - Assume $\ell > 4$ (otherwise, solve by brute force)

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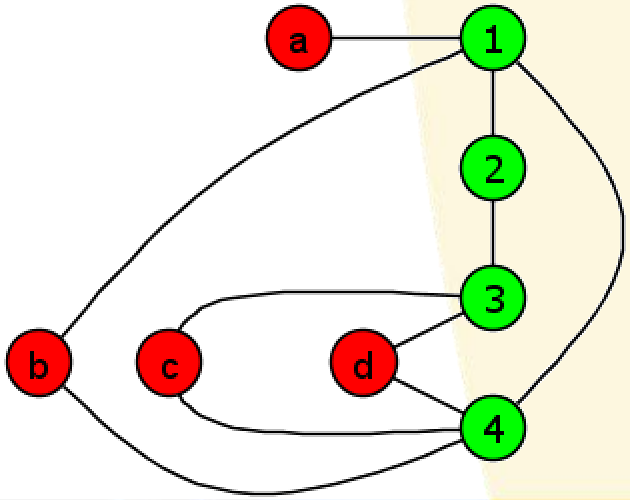
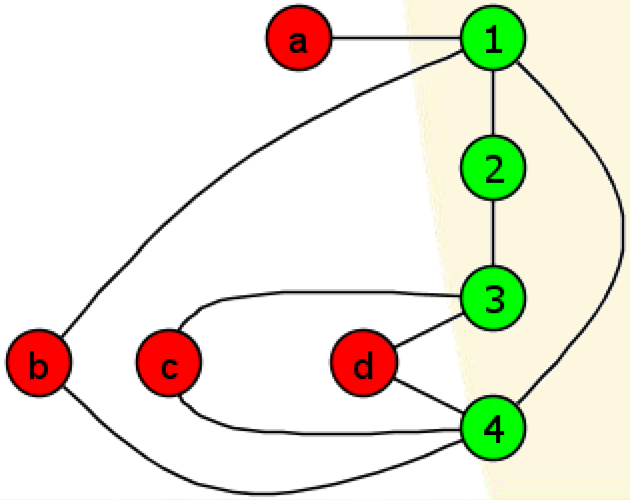


Reduction algorithm

- Bipartite auxiliary graph H = (R ∪ B, E)



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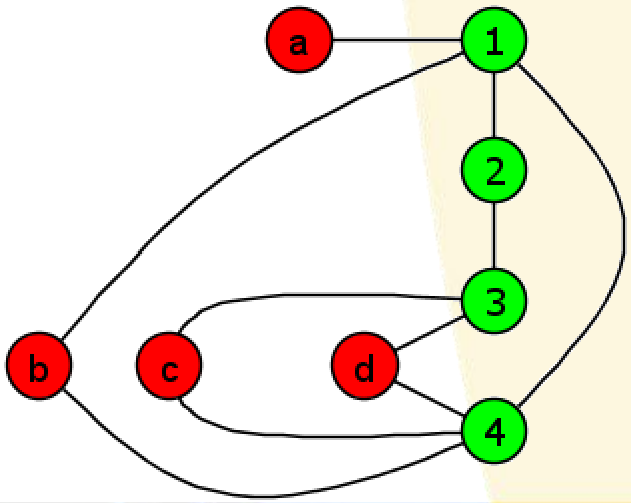

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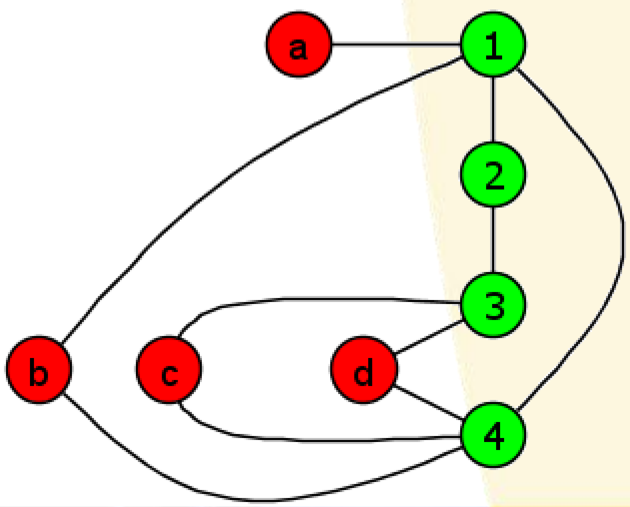

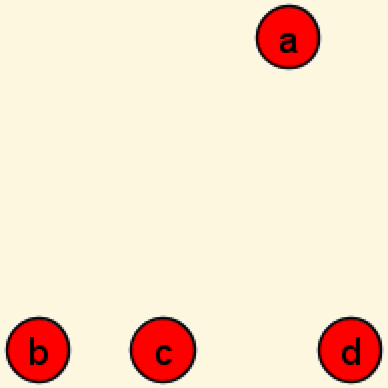

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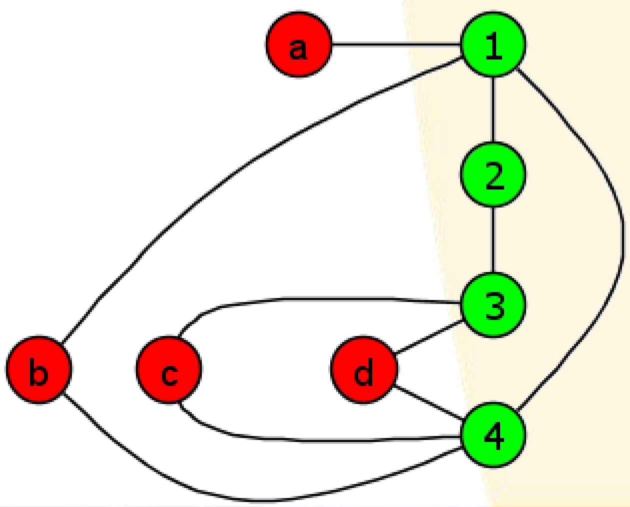

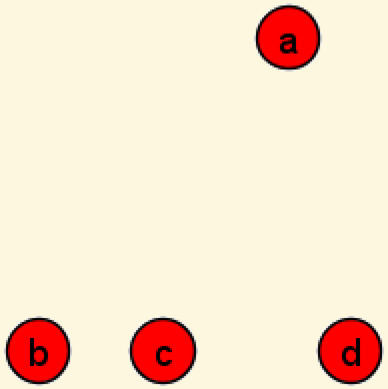

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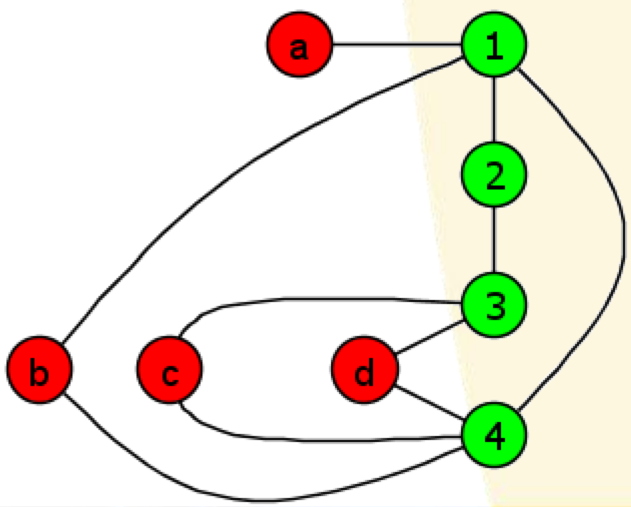

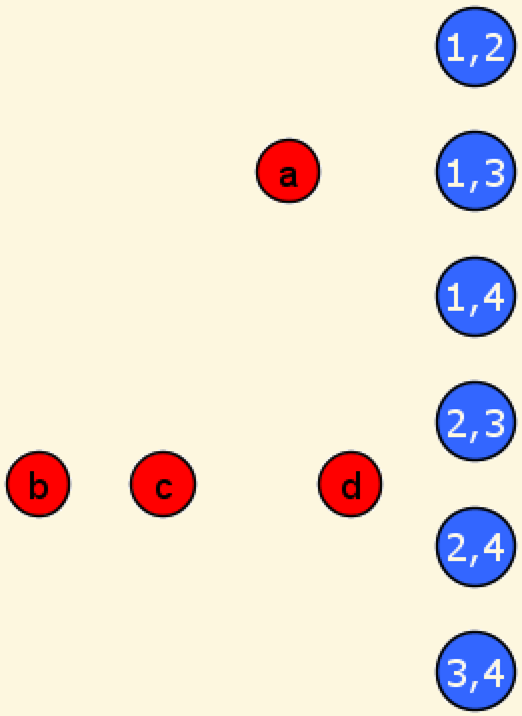

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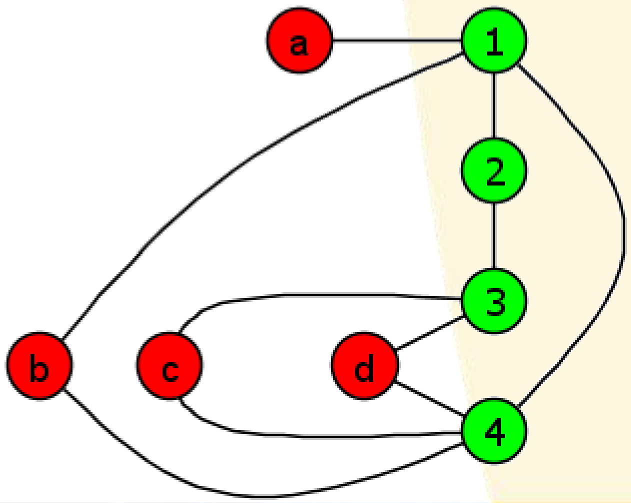

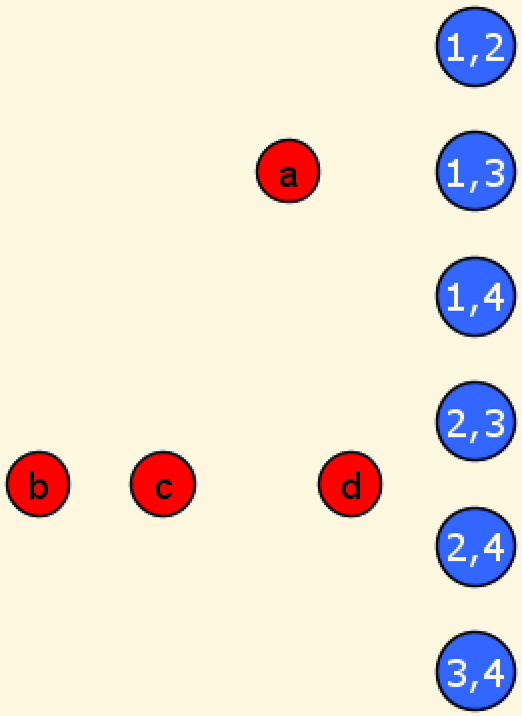

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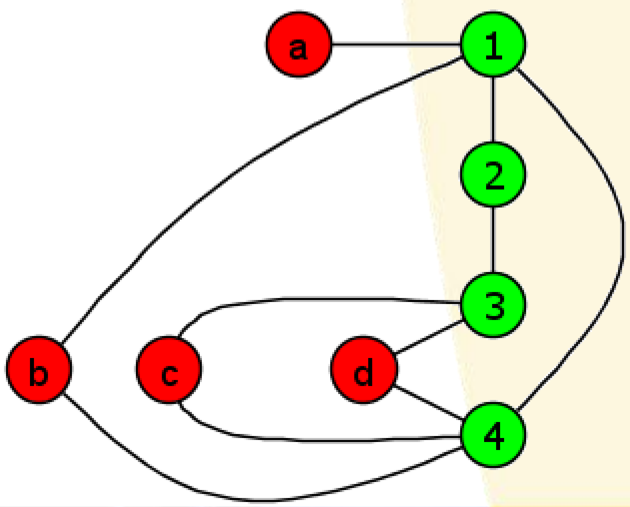

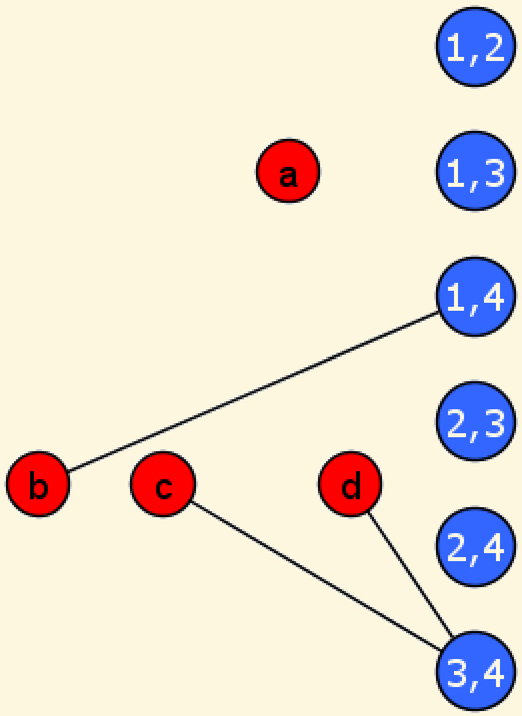

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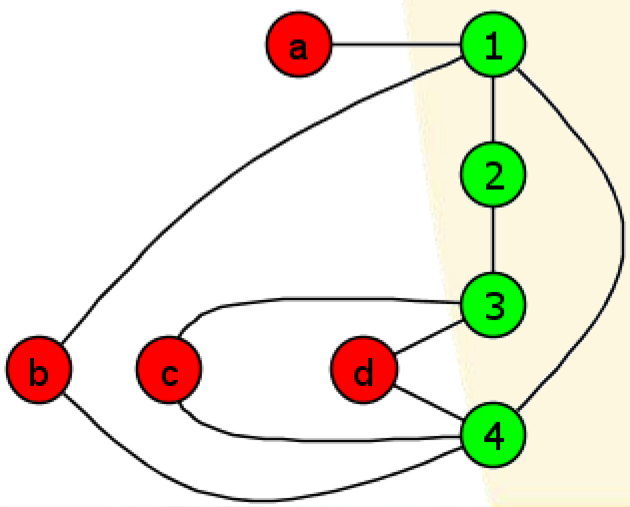

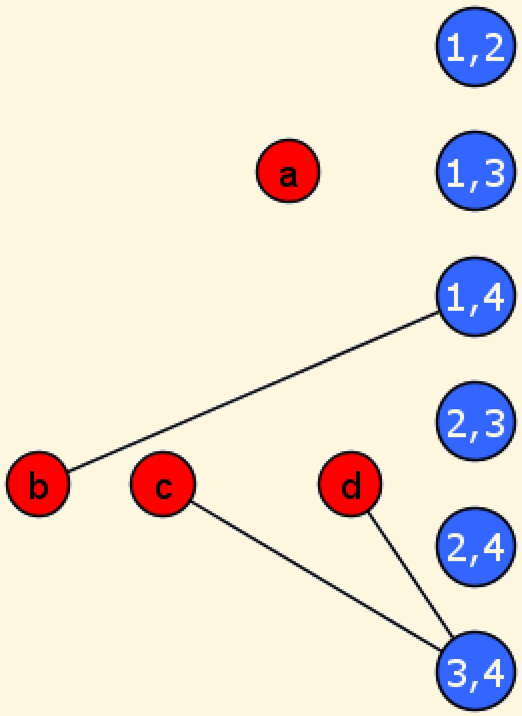

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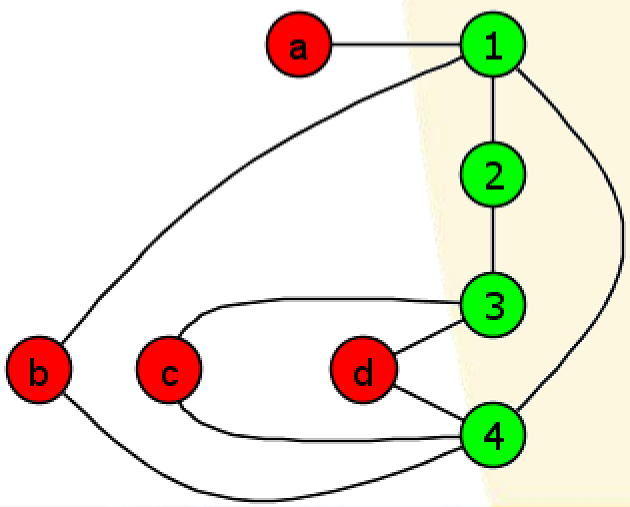

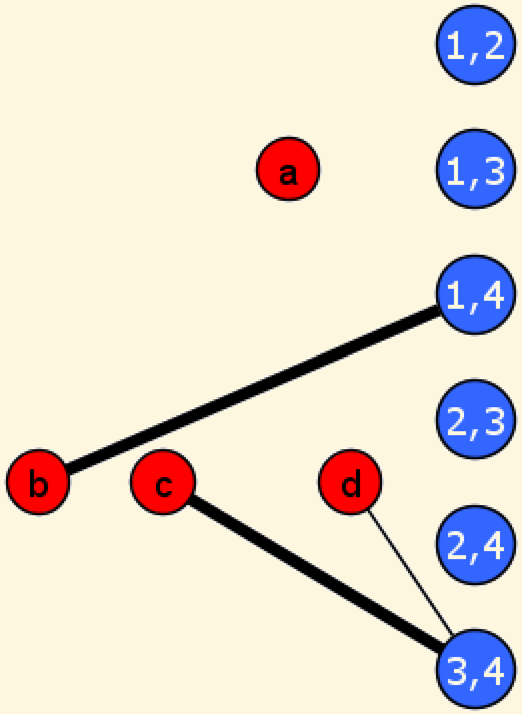

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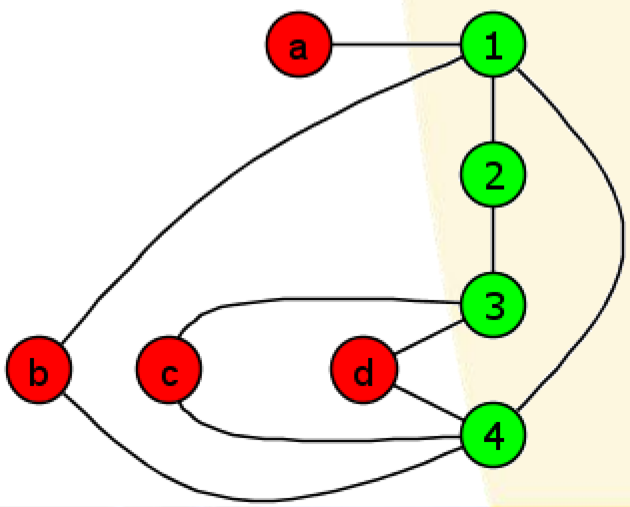
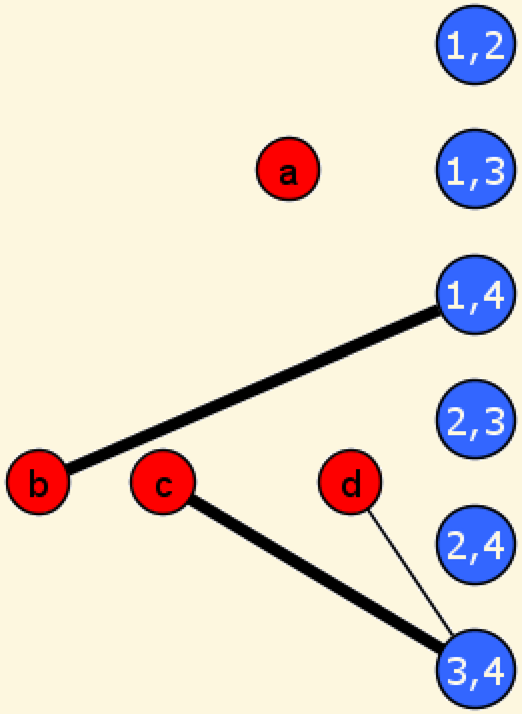
  - Let R_U be the unsaturated red vertices





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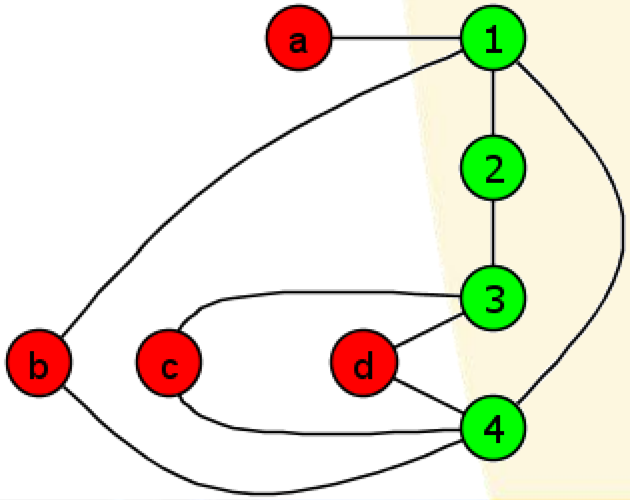

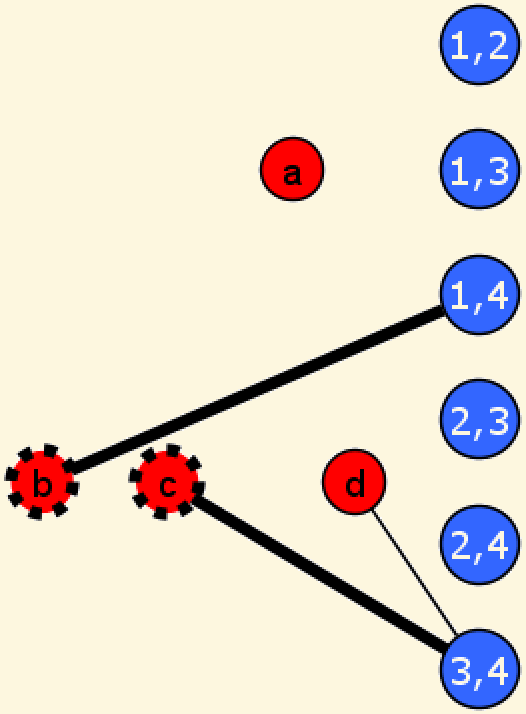

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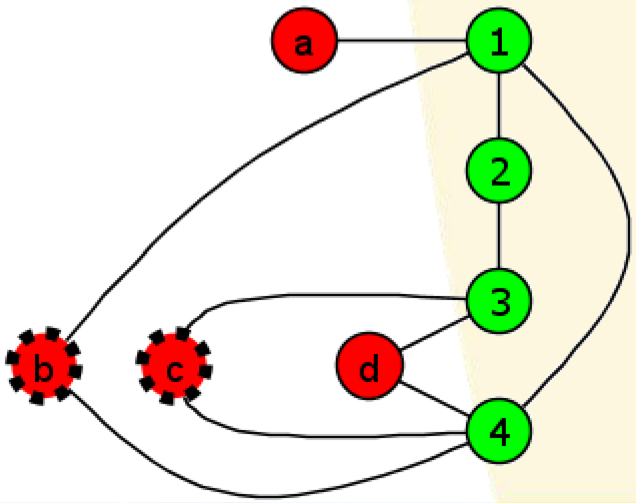
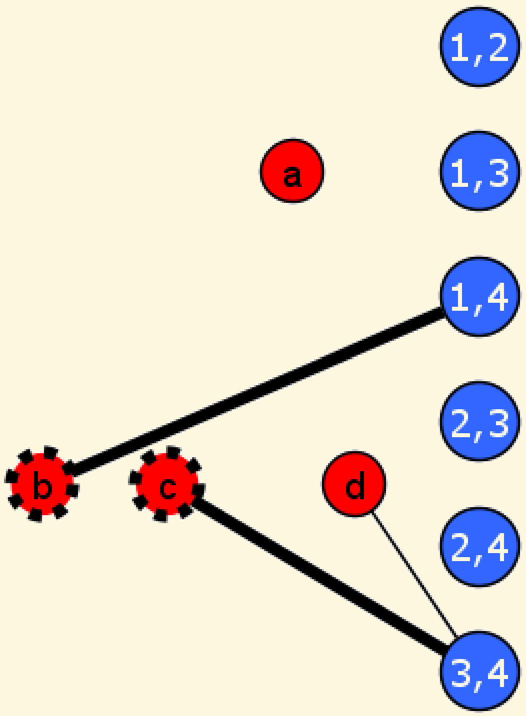
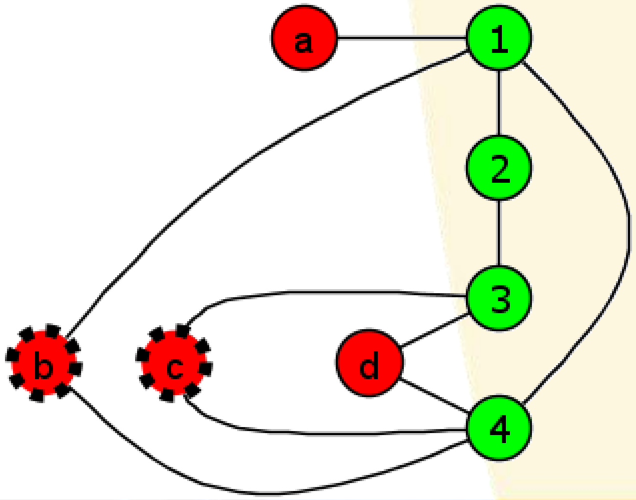
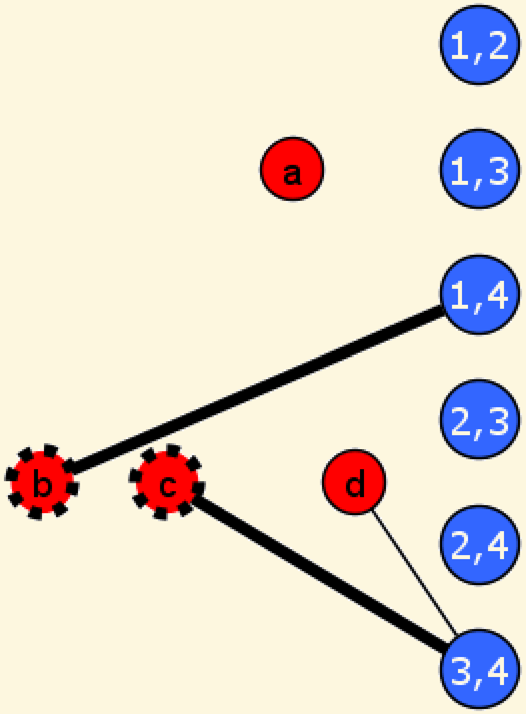




Reduction algorithm

- Bipartite auxiliary graph $H = (R \cup B, E)$
  - Red vertices are $V(G) \setminus X$
  - Blue vertex $v(p,q)$ for each pair $p,q \in X$
    - $v(p,q)$ adjacent to $N(p) \cap N(q) \setminus X$
- Compute maximum matching in H
  - Let R_U be the unsaturated red vertices
- Output $G - R_U$ with $\leq |X| + |X|^2$ vertices





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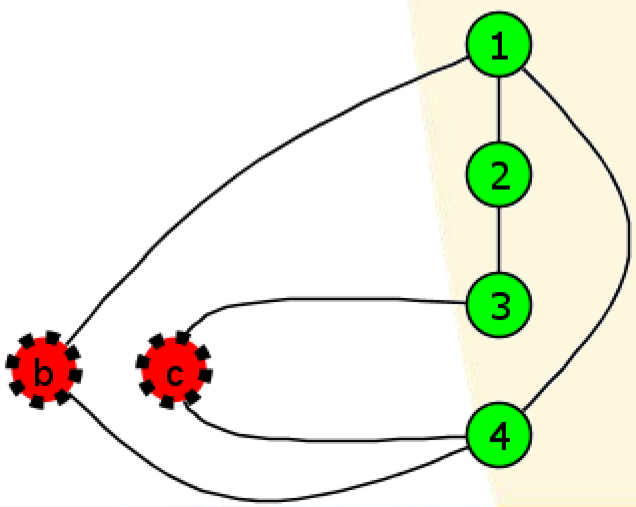

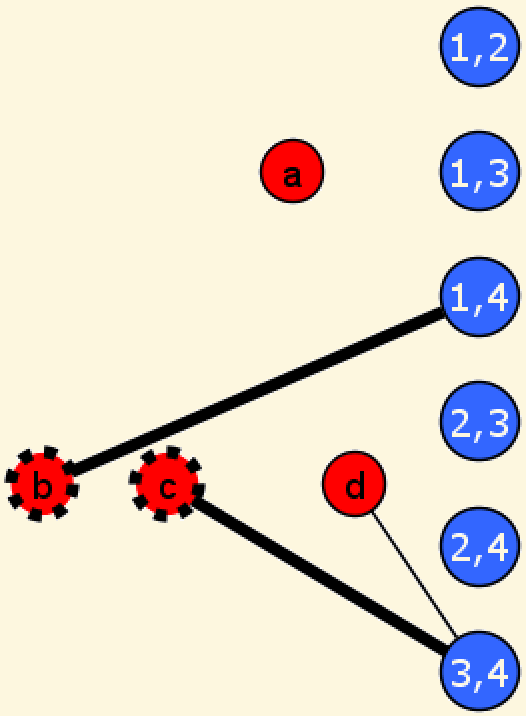

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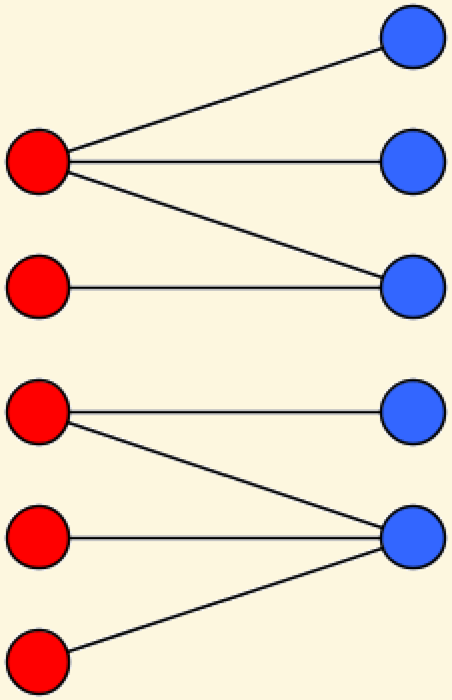

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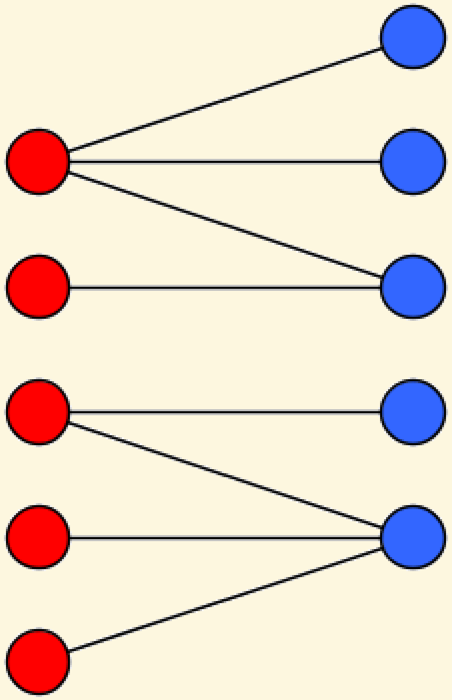
- Let M be a maximum matching in H



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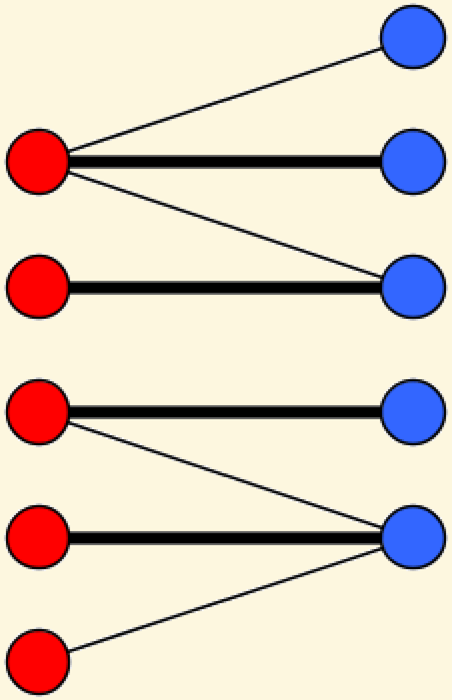
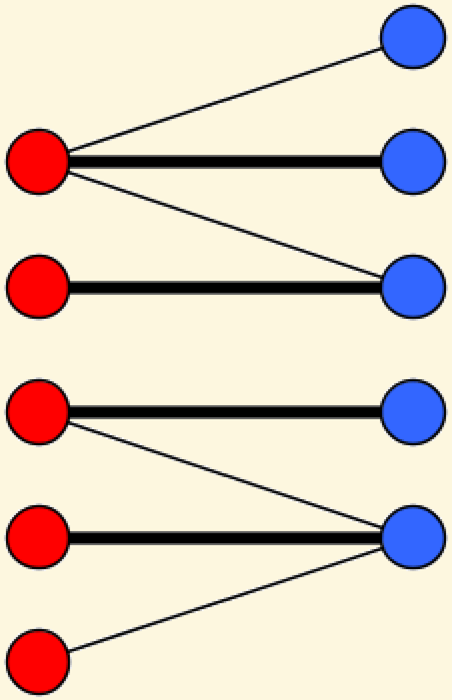


Property of maximum matchings

- Let H = (R ∪ B, E) be a bipartite graph
- Let M be a maximum matching in H
- Let R_U be vertices of R **not** saturated by M



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- Let H = (R ∪ B, E) be a bipartite graph
- Let M be a maximum matching in H
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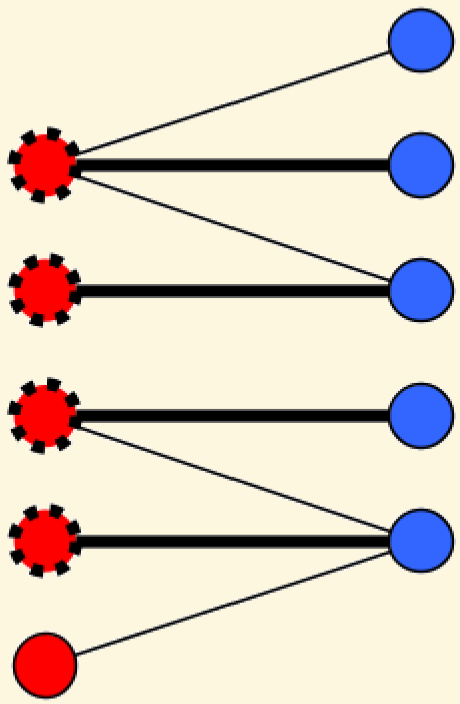

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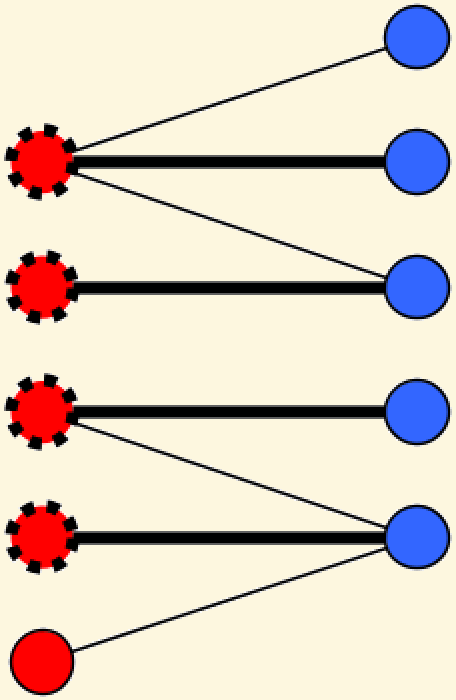

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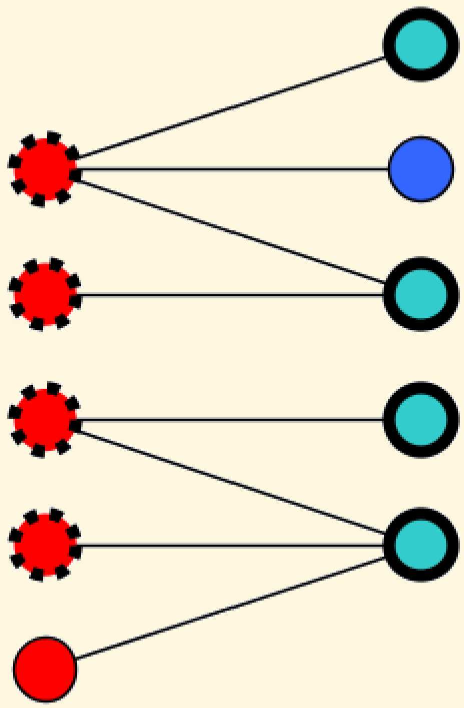

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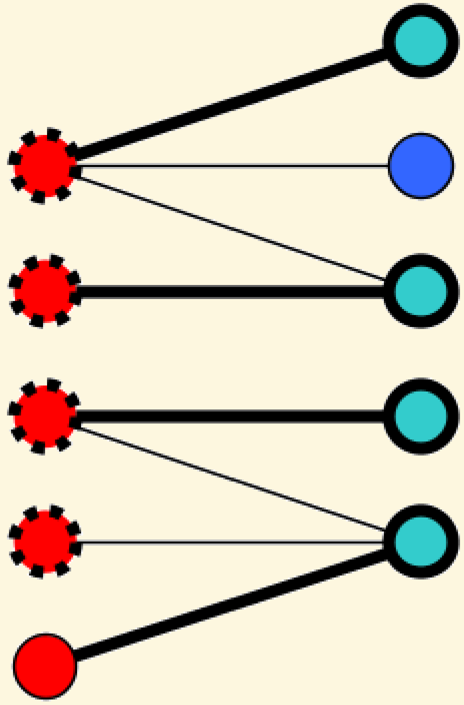

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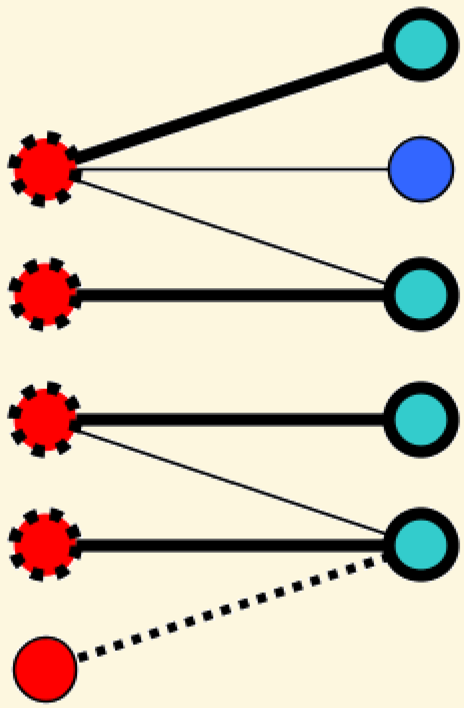

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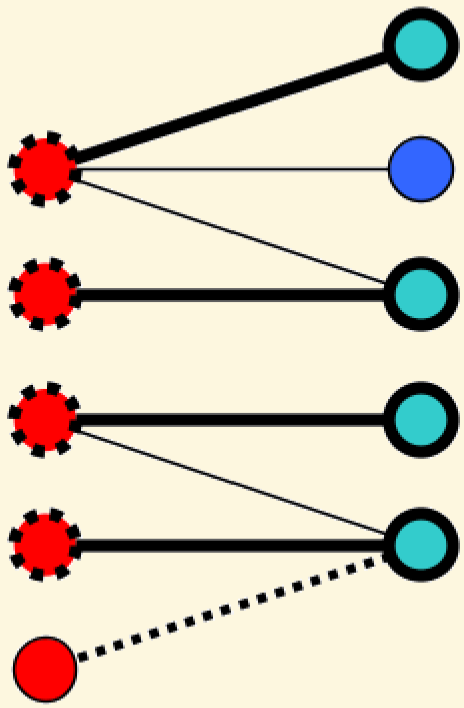

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- Proof using augmenting paths

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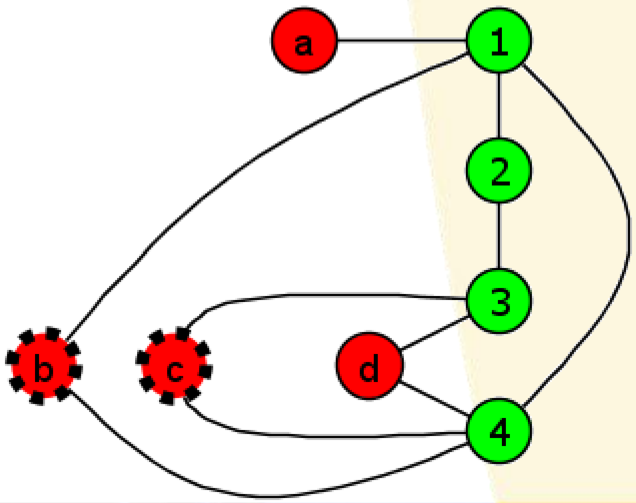

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- G has a cycle of length ℓ ⇔ G – R_U has a cycle of length ℓ

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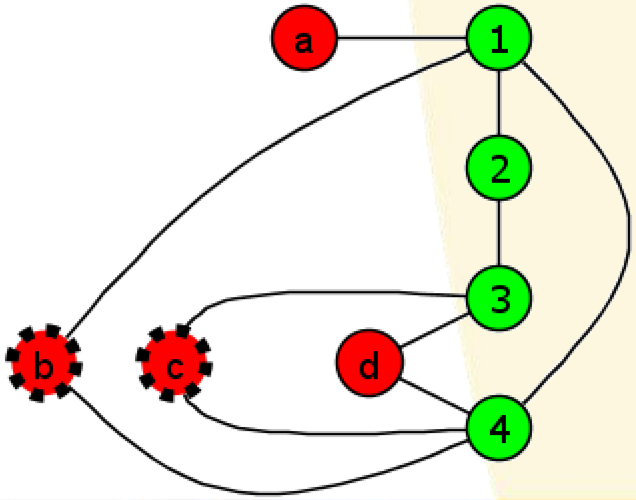

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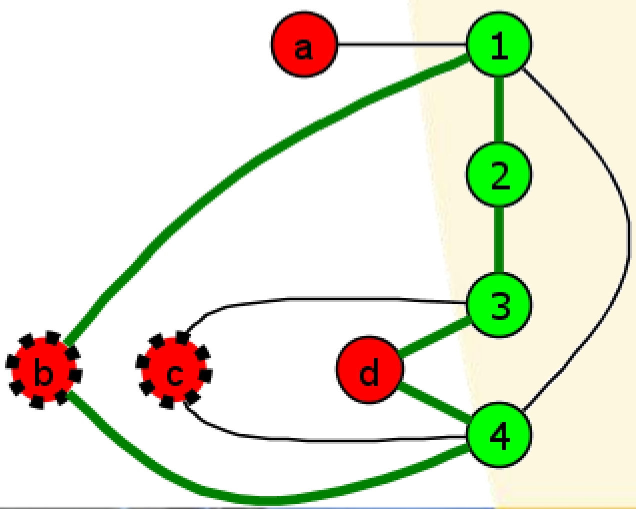

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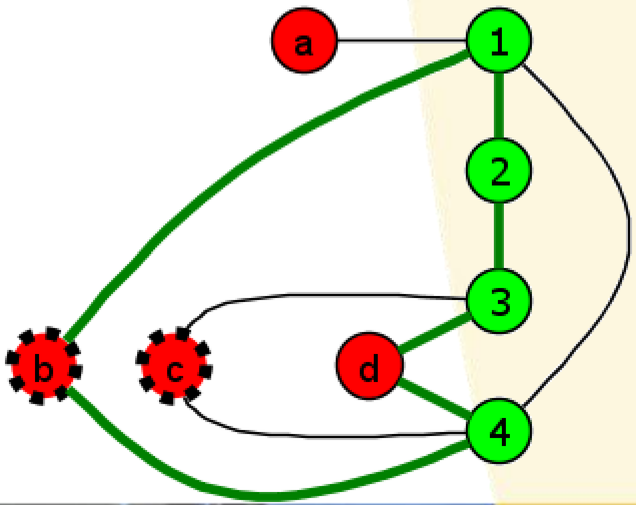

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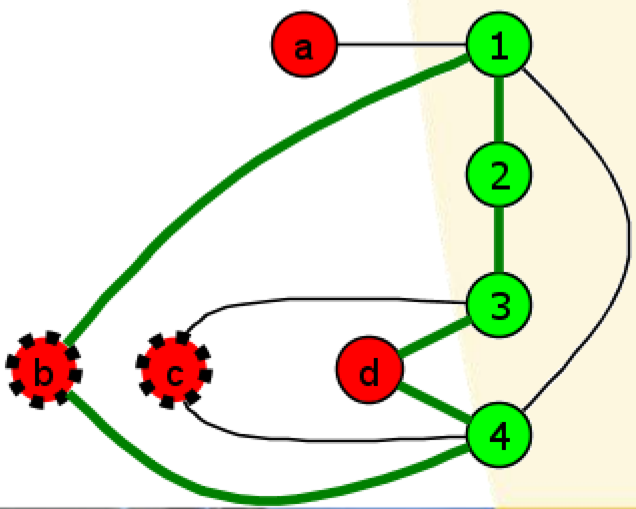

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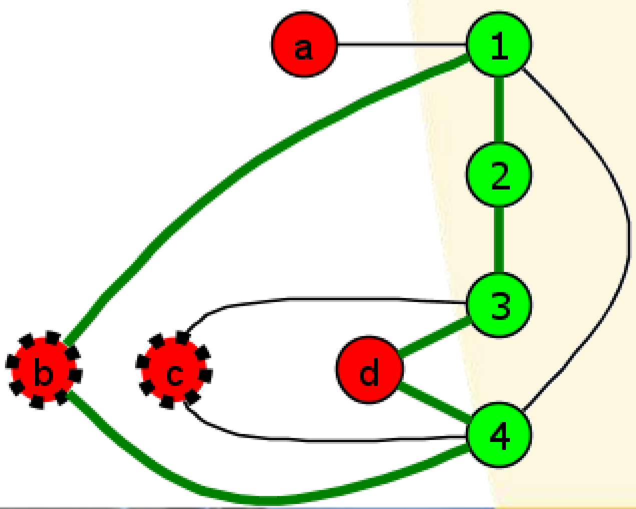

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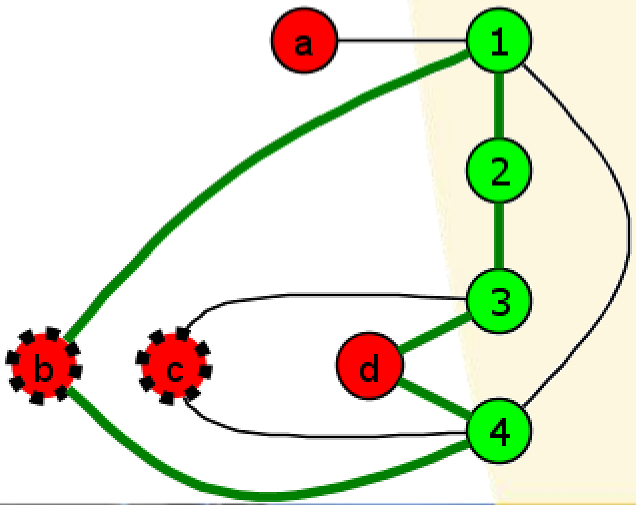

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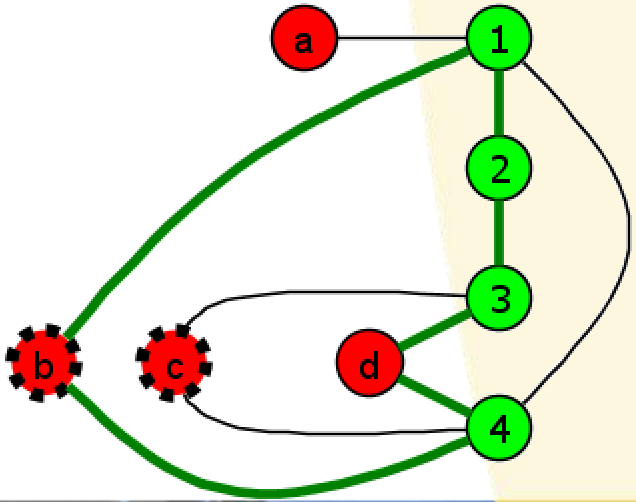

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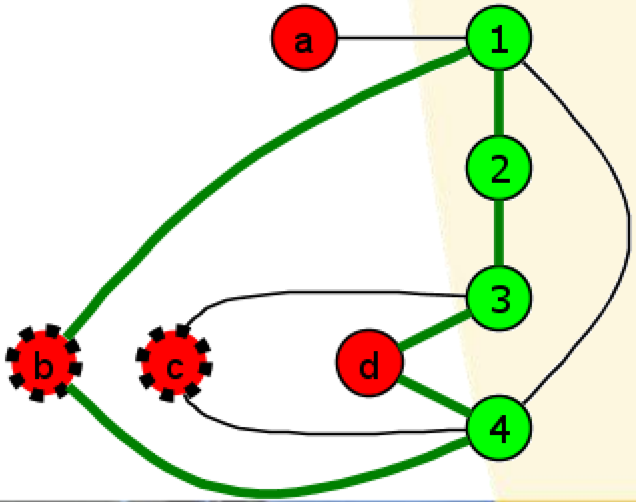

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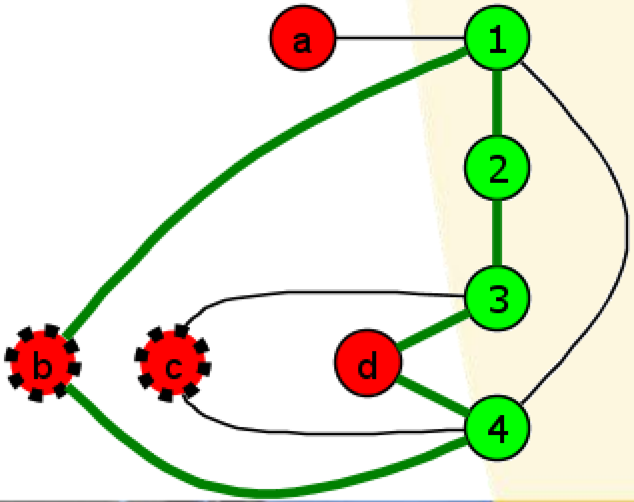

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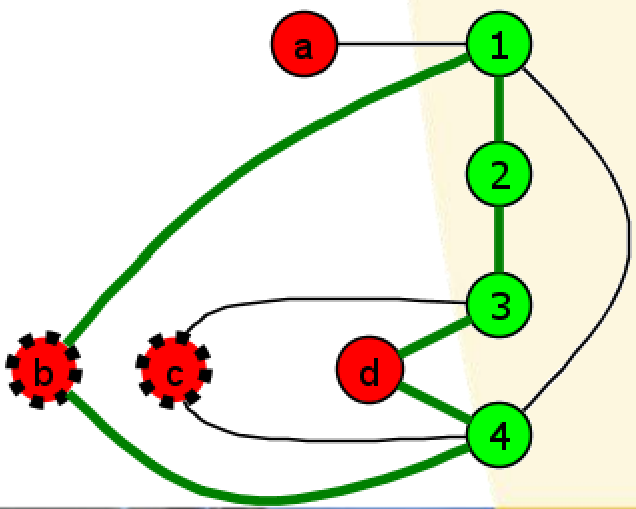

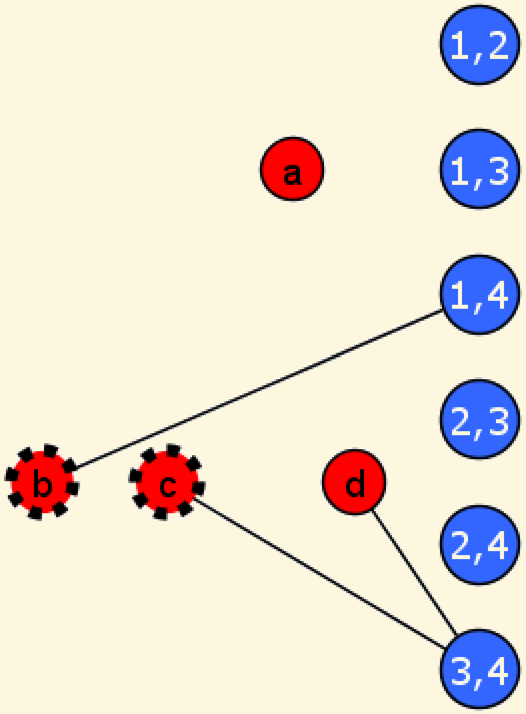

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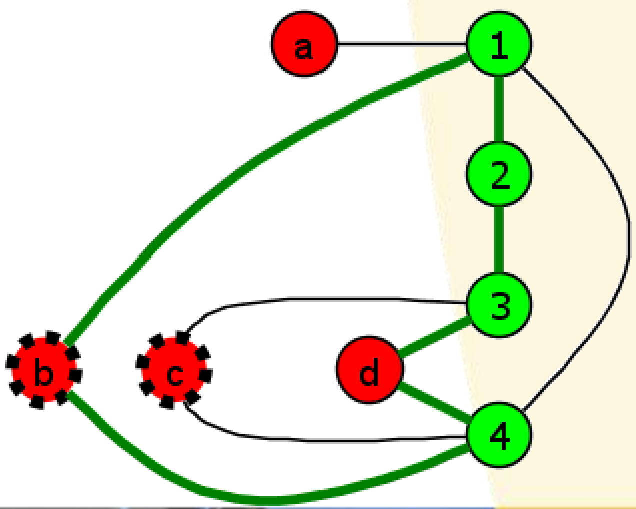

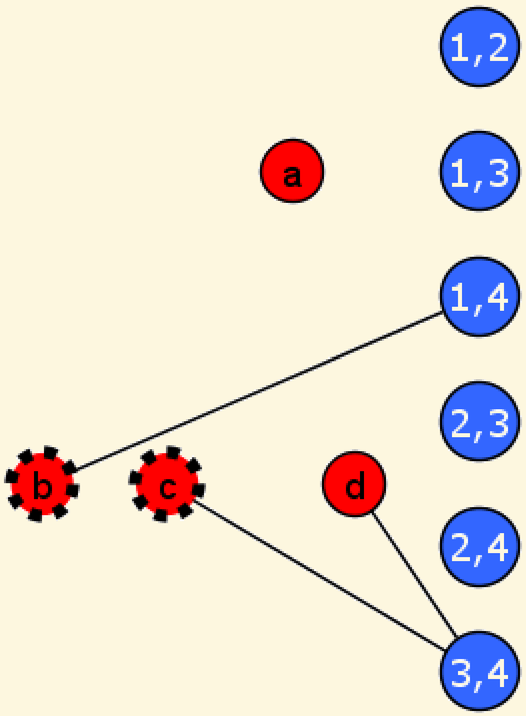

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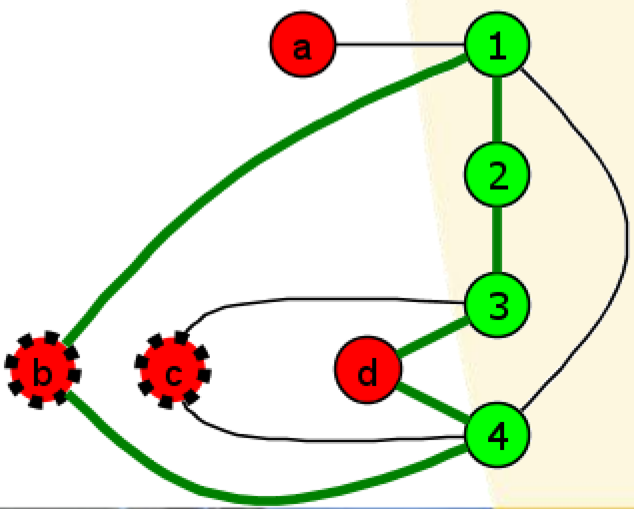

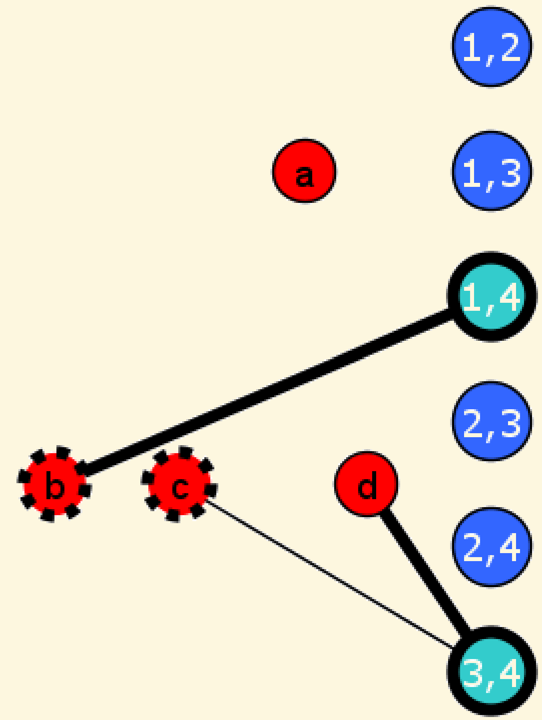

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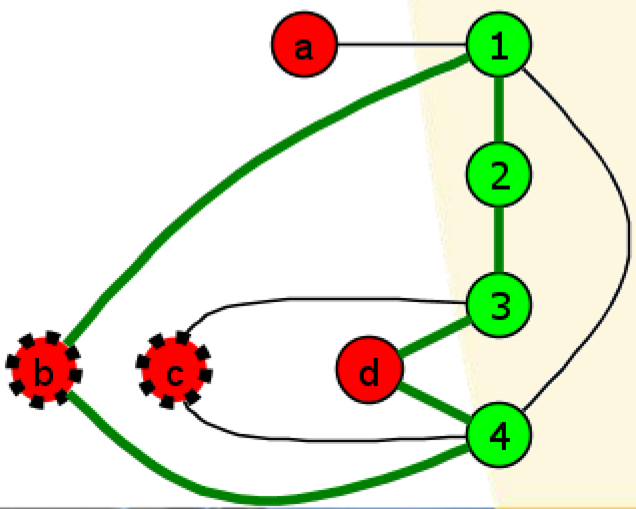

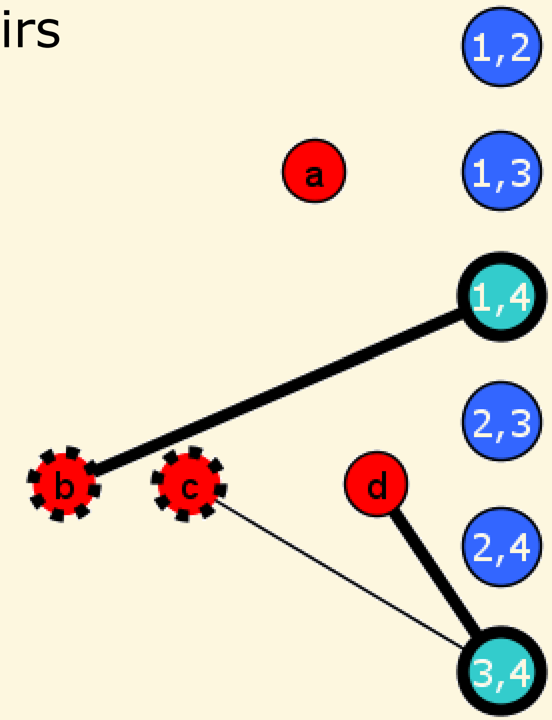

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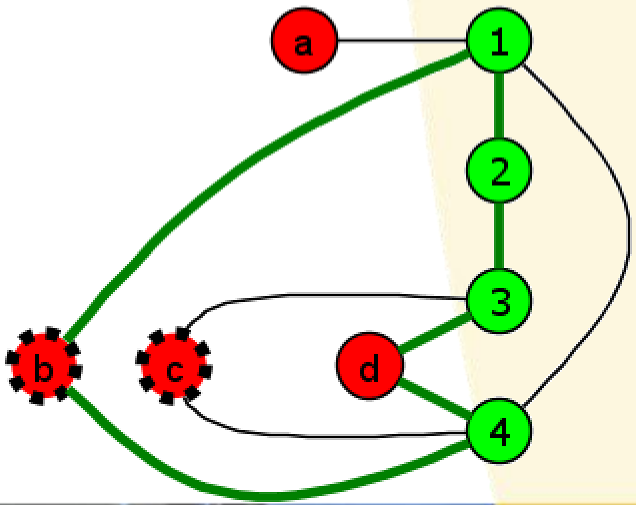

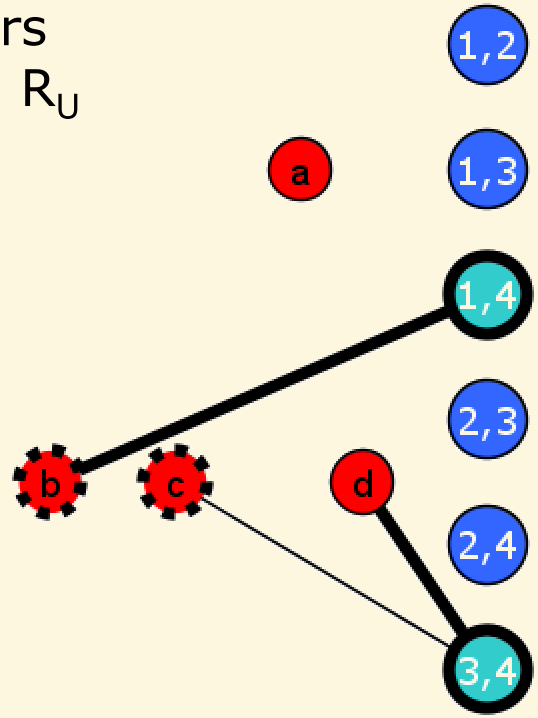

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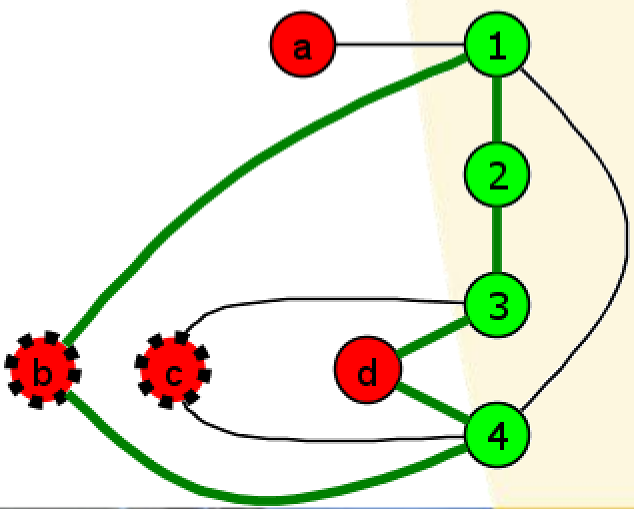

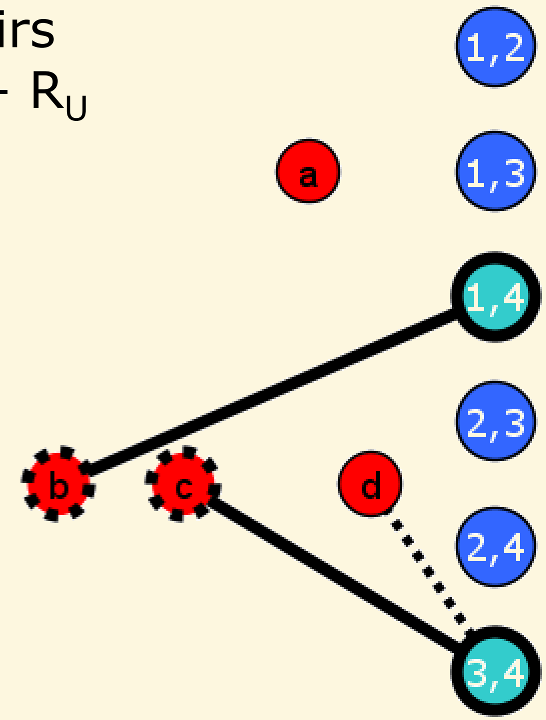

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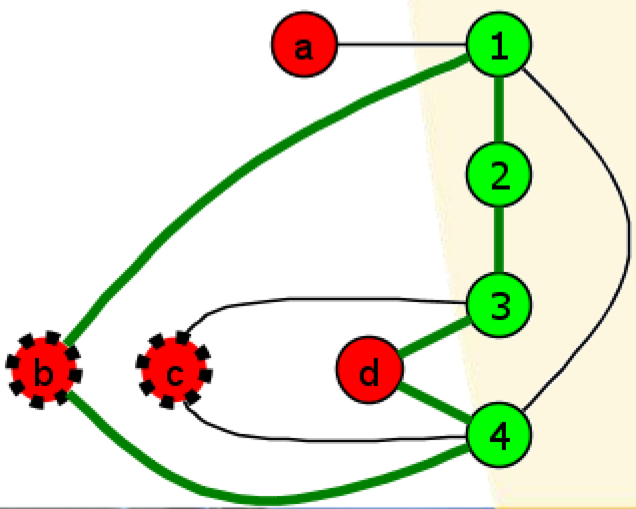

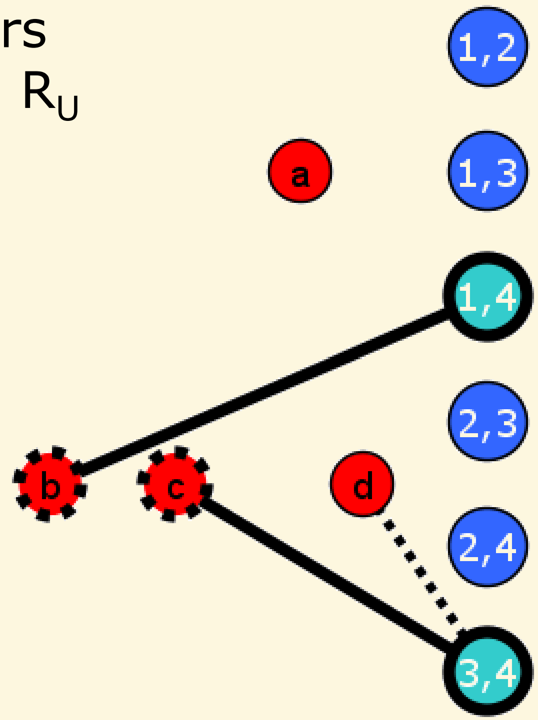

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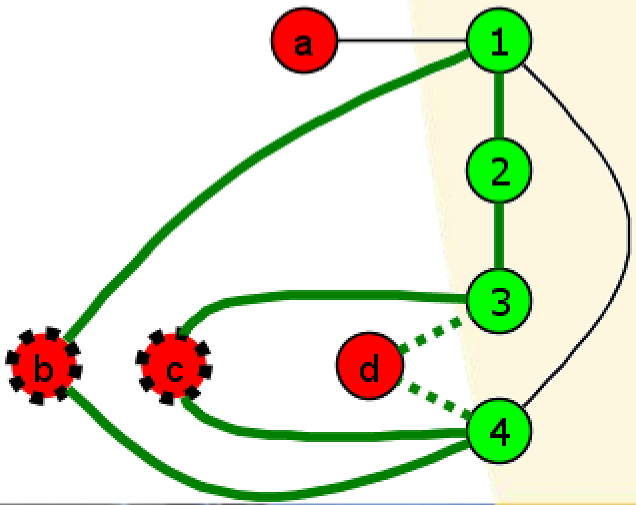

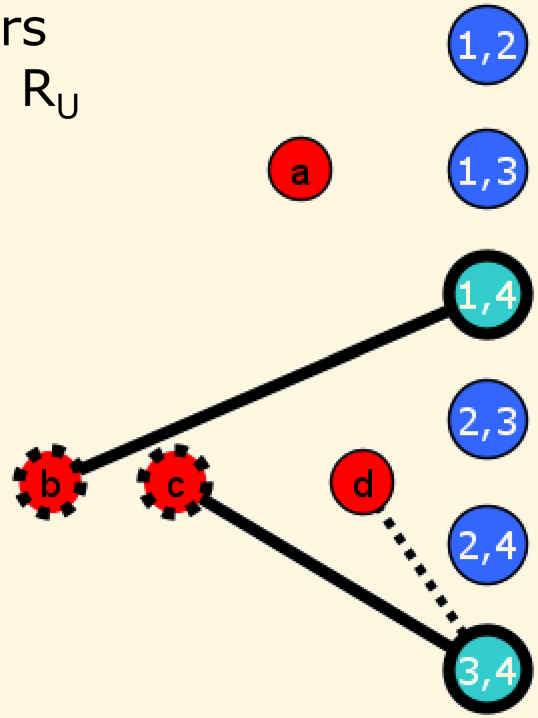

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- Also applies to Long Path, Disjoint Paths, Disjoint Cycles

Polynomial kernel by Max Leaf Number

LONG CYCLE

Long Cycle parameterized by Max Leaf Number

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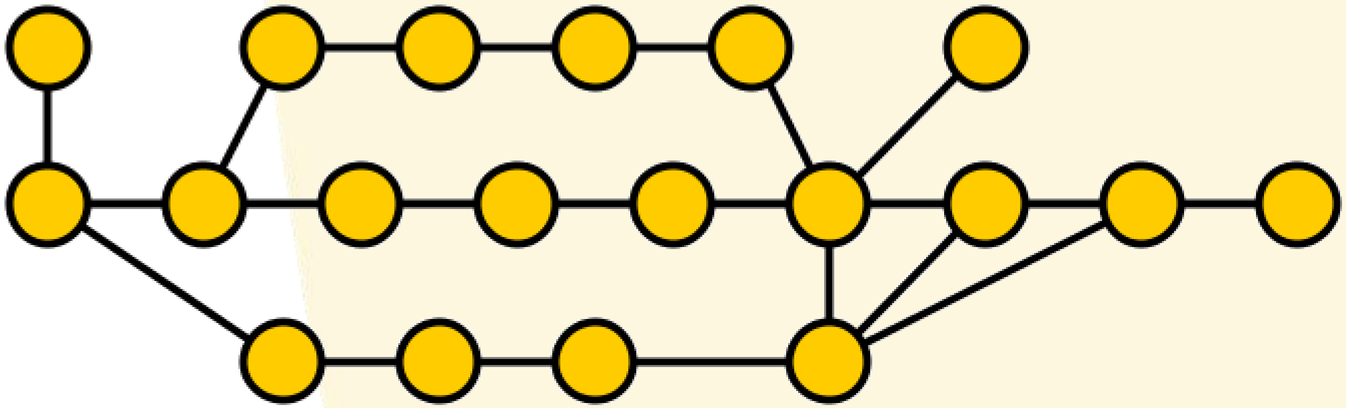

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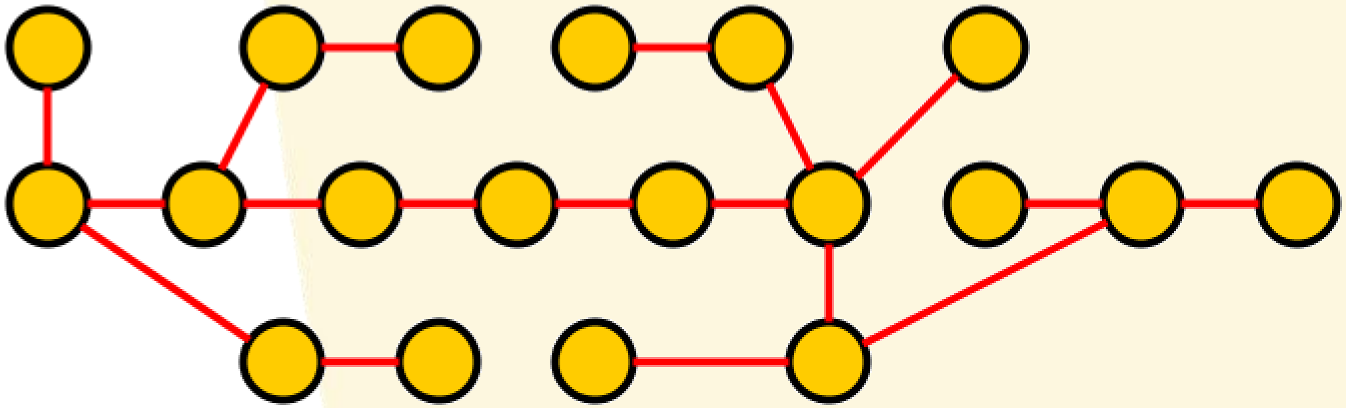

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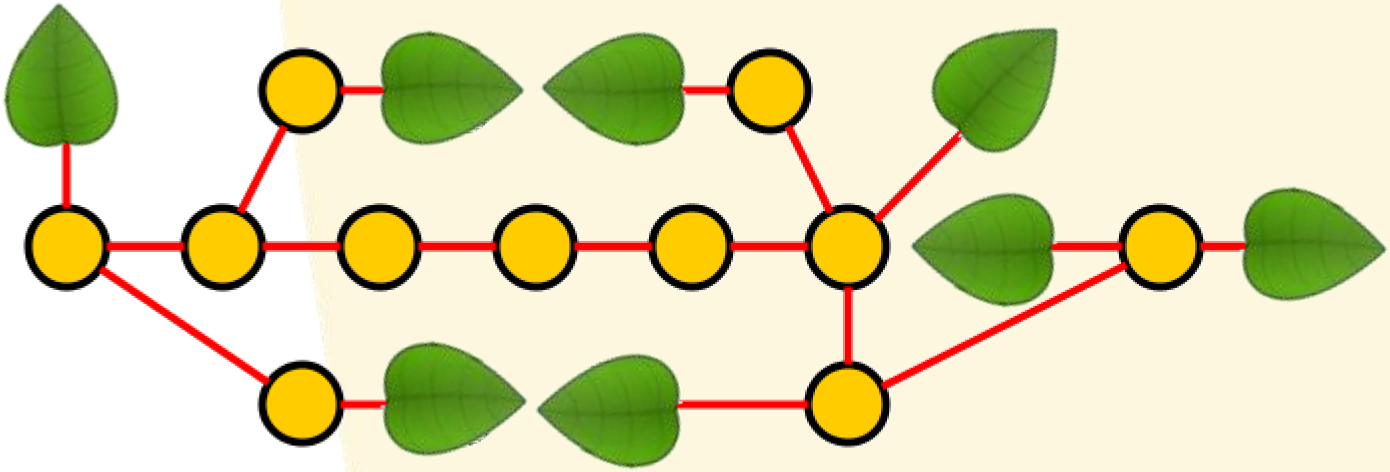

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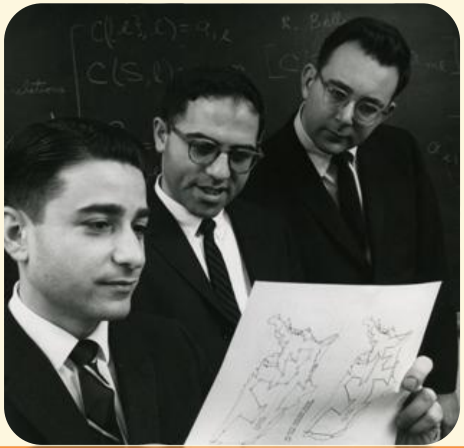

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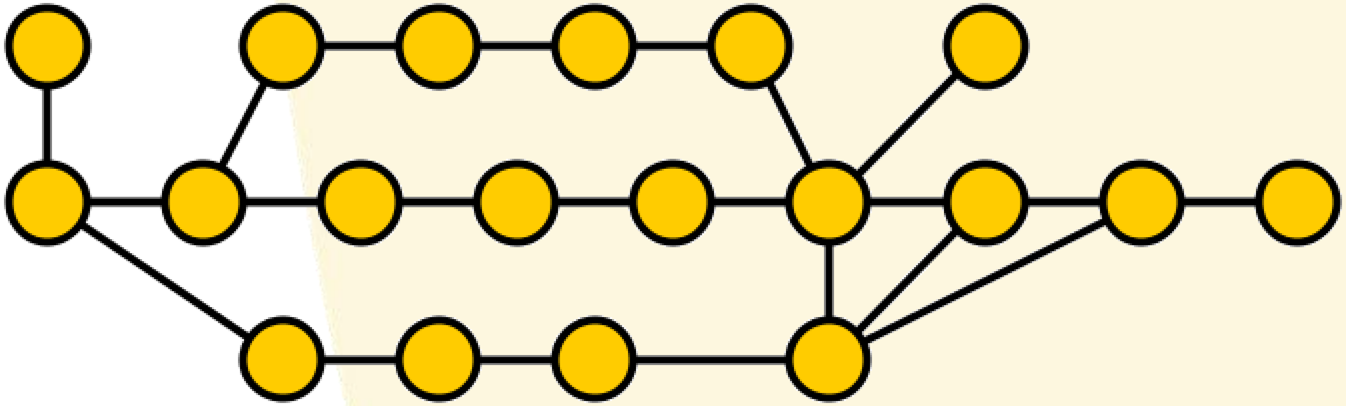
2. Held-Karp Dynamic Programming

3. Karp Reduction

The kernelization algorithm



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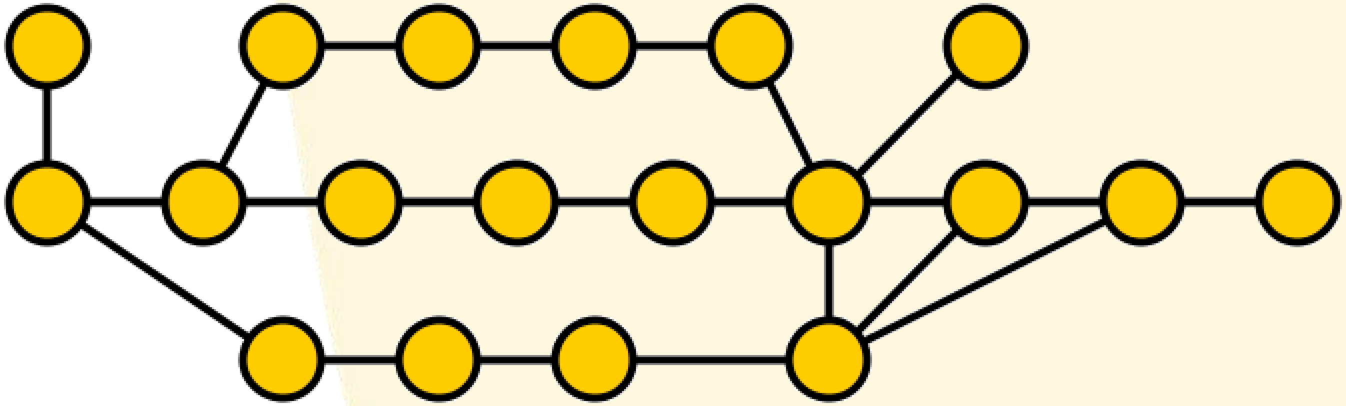

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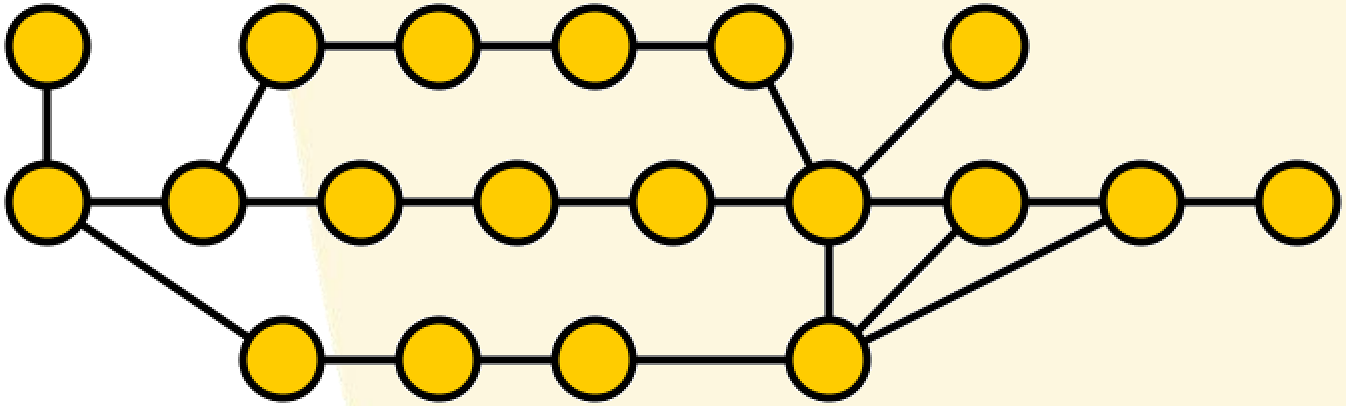

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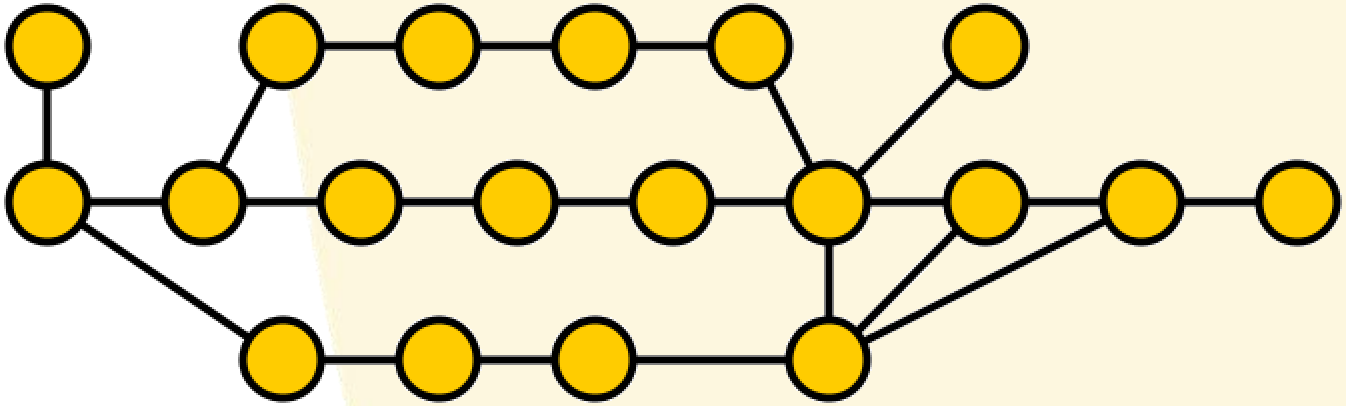

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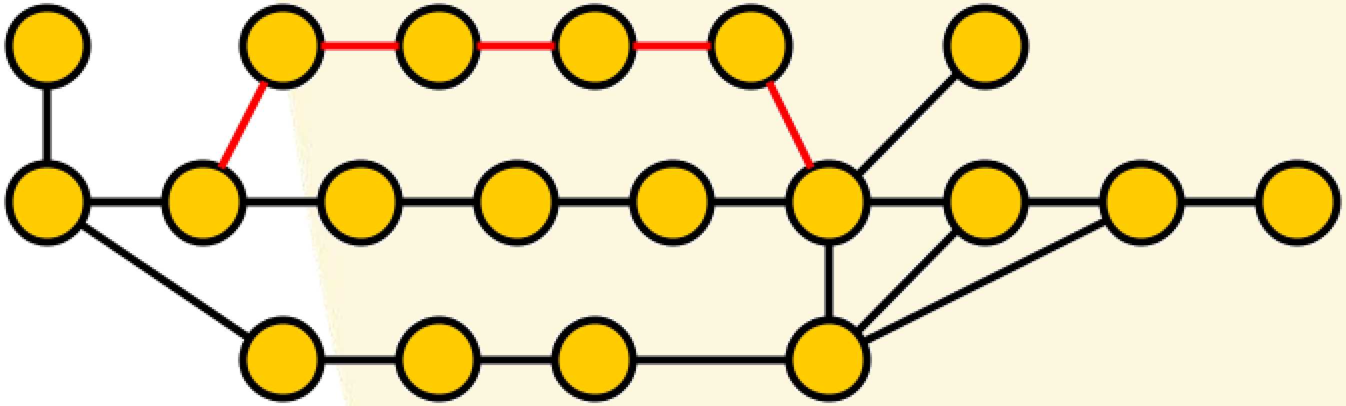

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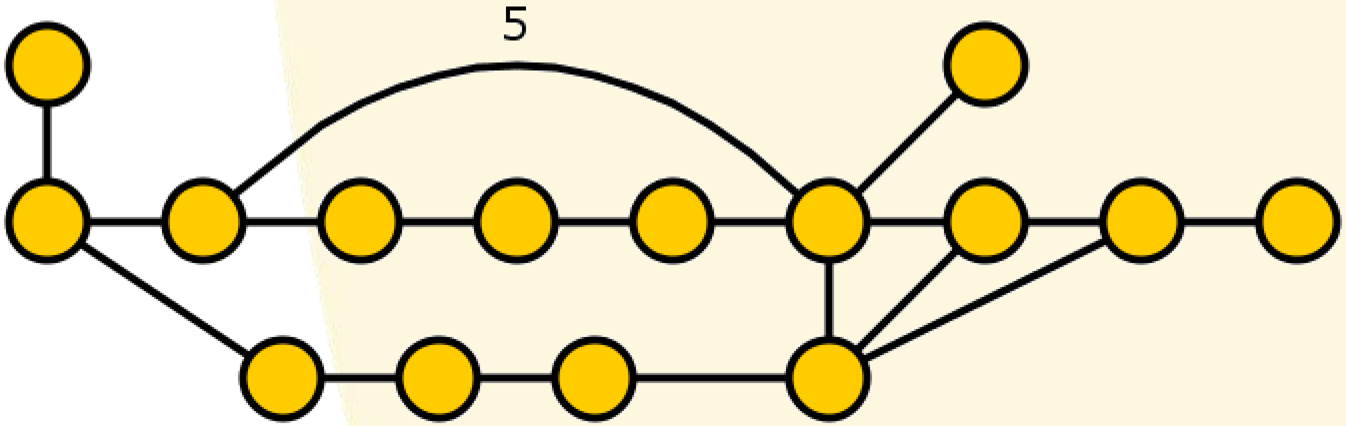

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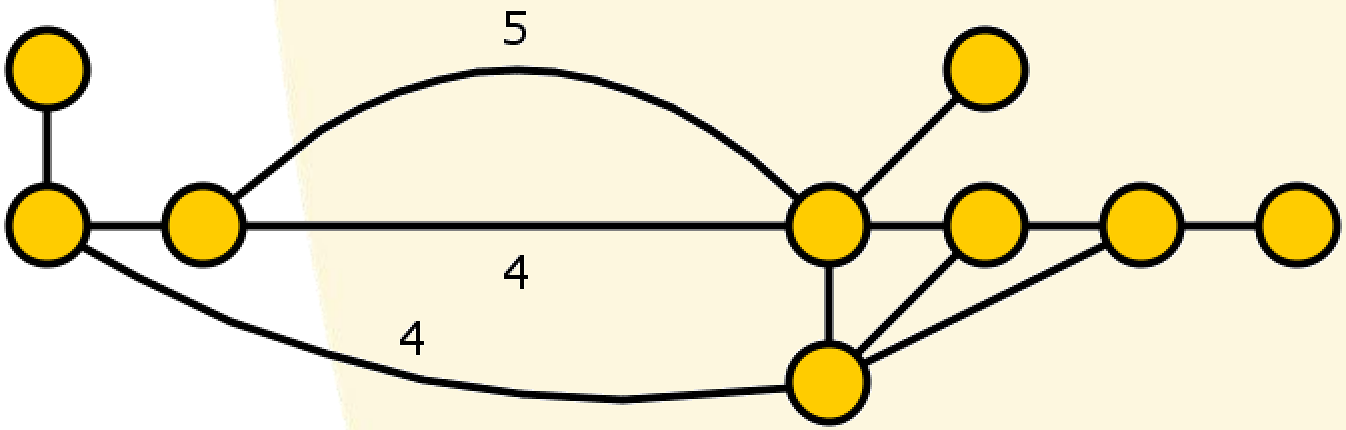

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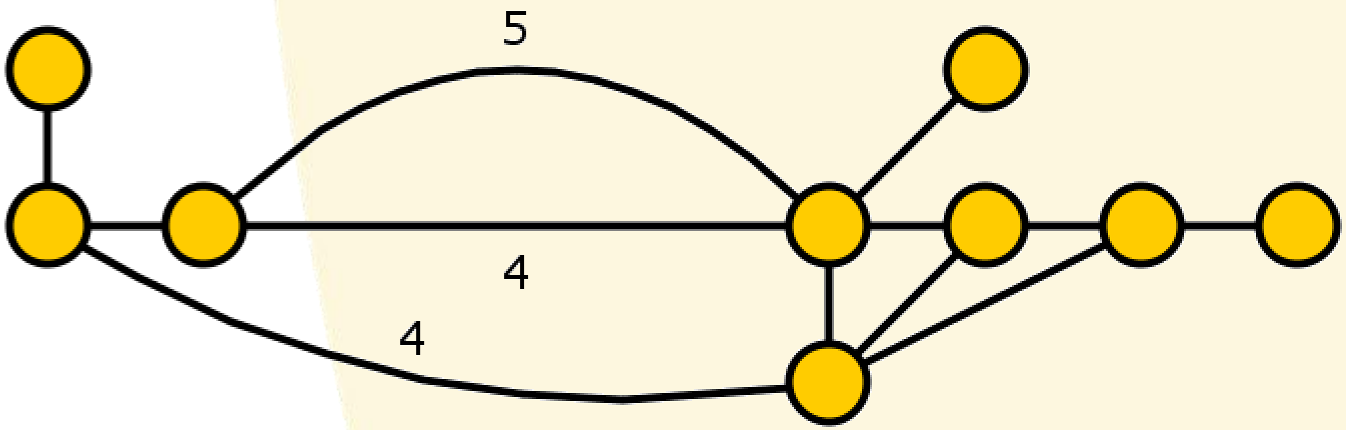

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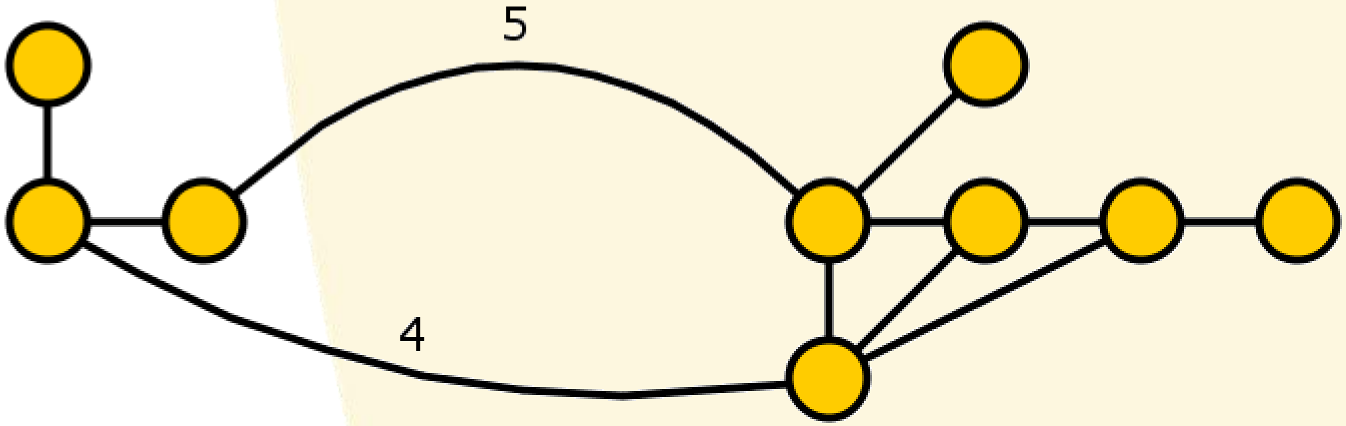

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- Reduce to weighted simple graph (G', w') with $|V(G')| = |X| \leq c \cdot k$

Held-Karp Dynamic Programming

- If binary encoding of a weight uses > $c \cdot k$ bits:
  - There were > $2^{c \cdot k}$ degree-2 vertices so $n > 2^{c \cdot k}$
  - Solve weighted instance: $O(2^{|X|} |X|^3)$ is $O(n^4)$ time

The kernelization algorithm

Kleitman-West Theorem

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Karp Reduction

- If binary encoding is small: (G', w', ℓ') has bitsize poly(k)
  - Weighted Long Cycle is in NP
  - Reduce to back to unweighted problem
  - Polynomial-time transformation, output has size poly(k)

DISCUSSION & CONCLUSION

Structural parameterizations of Hamiltonian Cycle (& related)

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Vertex Cover Number

- Deletion distance to treewidth 0

Structural parameterizations of Hamiltonian Cycle (& related)

Vertex Cover Number

- Deletion distance to treewidth 0

Structural parameterizations of Hamiltonian Cycle (& related)

Vertex Cover Number

- Deletion distance to treewidth 0

Deletion distance to Outerplanar

- Deletion distance to treewidth 2

Structural parameterizations of Hamiltonian Cycle (& related)

Vertex Cover Number

- Deletion distance to treewidth 0

Deletion distance to Outerplanar

- Deletion distance to treewidth 2

Structural parameterizations of Hamiltonian Cycle (& related)

Vertex Cover Number

- Deletion distance to treewidth 0

Feedback Vertex Number

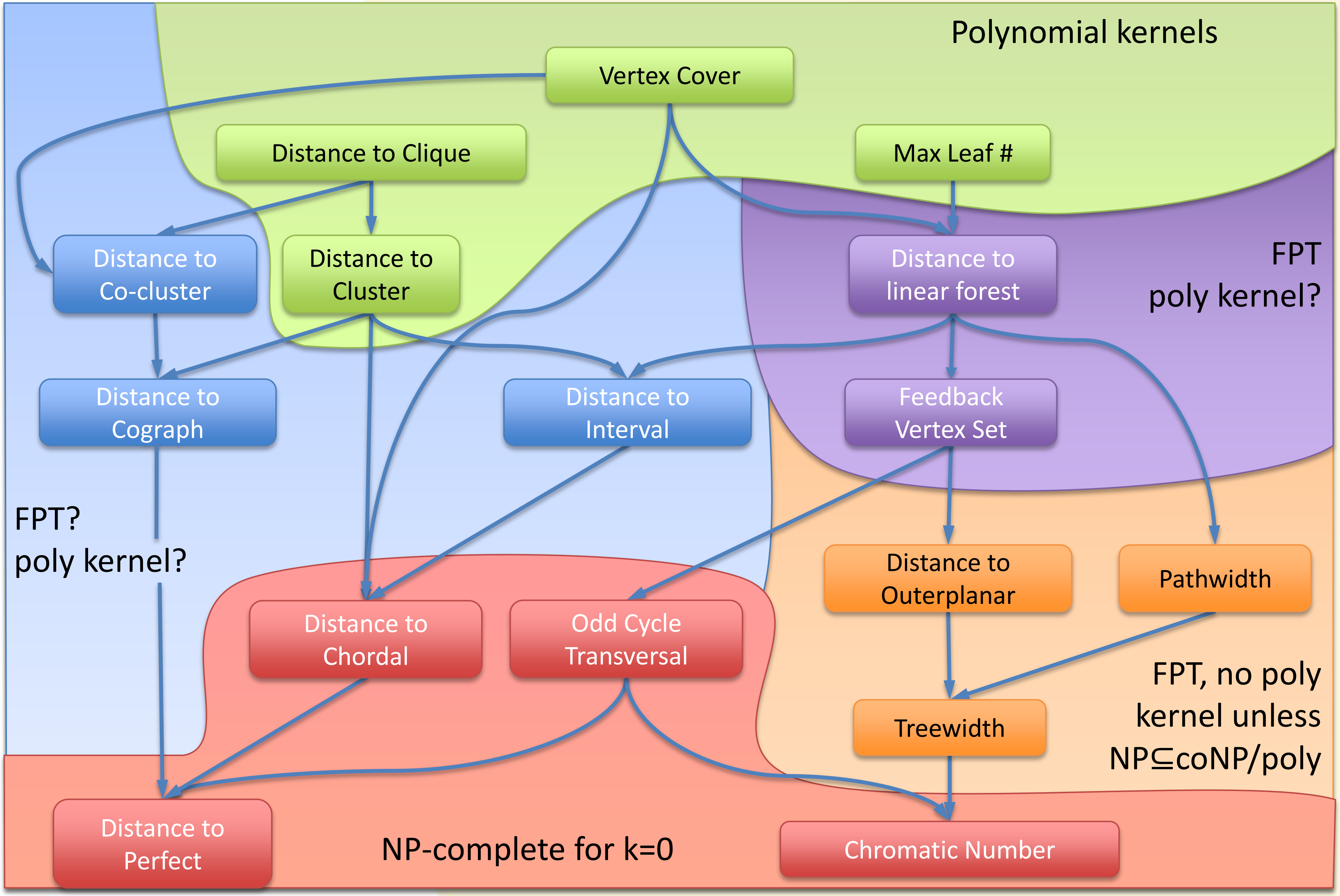
- Deletion distance to treewidth 1

Deletion distance to Outerplanar

- Deletion distance to treewidth 2

Structural parameterizations of Hamiltonian Cycle (& related)

Vertex Cover Number	Feedback Vertex Number	Deletion distance to Outerplanar
• Deletion distance to treewidth 0	• Deletion distance to treewidth 1	• Deletion distance to treewidth 2



Complexity overview for Long Cycle parameterized by…

Conclusion

- Structural parameterizations of Path and Cycle problems admit polynomial kernels
- Various upper and lower-bound results

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- Structural parameterizations of Path and Cycle problems admit polynomial kernels
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Poly kernels for Long Path parameterized by:

- feedback vertex number
- vertex-deletion distance to a cograph

Poly kernels for Long Path parameterized by:

- Max Leaf Number, *without* using binary encoding?

Is Longest Path in FPT …

- parameterized by a (given) deletion set to an Interval graph?

Conclusion

- Structural parameterizations of Path and Cycle problems admit polynomial kernels
- Various upper and lower-bound results

Poly kernels for Long Path parameterized by:

encou...

Is Longest Path in FPT …

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THANK YOU!